

A Simple Method for Measuring Stiffness During Running

Jean-Benoît Morin¹, Georges Dalleau², Heikki Kyröläinen³,
Thibault Jeannin¹, and Alain Belli¹

¹University of Saint-Etienne; ²University of La Réunion
³University of Jyväskylä

The spring-mass model, representing a runner as a point mass supported by a single linear leg spring, has been a widely used concept in studies on running and bouncing mechanics. However, the measurement of leg and vertical stiffness has previously required force platforms and high-speed kinematic measurement systems that are costly and difficult to handle in field conditions. We propose a new “sine-wave” method for measuring stiffness during running. Based on the modeling of the force-time curve by a sine function, this method allows leg and vertical stiffness to be estimated from just a few simple mechanical parameters: body mass, forward velocity, leg length, flight time, and contact time. We compared this method to force-platform-derived stiffness measurements for treadmill dynamometer and overground running conditions, at velocities ranging from 3.33 m·s⁻¹ to maximal running velocity in both recreational and highly trained runners. Stiffness values calculated with the proposed method ranged from 0.67% to 6.93% less than the force platform method, and thus were judged to be acceptable. Furthermore, significant linear regressions ($p < 0.01$) close to the identity line were obtained between force platform and sine-wave model values of stiffness. Given the limits inherent in the use of the spring-mass model, it was concluded that this sine-wave method allows leg and stiffness estimates in running on the basis of a few mechanical parameters, and could be useful in further field measurements.

Key Words: spring-mass, modeling, practical calculation

During running, the musculoskeletal structures of the legs alternately store and return elastic energy, so the legs can be described as springs loaded by the runner’s body mass, constituting the “spring-mass model” (Alexander, 1992; Blickhan, 1989; Cavagna, Heglund, & Taylor, 1977; Cavagna, Saibene, & Margaria,

¹Physiology Laboratory, PPEH Res. Unit, Univ. of Saint-Etienne, CHU Bellevue, 42055 St-Etienne Cedex 02, France; ²Sport Sciences Res. Center, Faculty of Science and Technology, Univ. of La Réunion, 117 General Ailleret St., 97430 Le Tampon, France; ³Neuromuscular Res. Center, Univ. of Jyväskylä, PO Box 35, 40014 Jyväskylä, Finland.

1964; McMahon & Cheng, 1990). This model has been a widely used concept for describing and studying the storage and return of elastic energy in the lower limbs in humans and other animals during bouncing and running (Dalleau, Belli, Bourdin, & Lacour, 1998; Farley, Blickhan, Saito, & Taylor, 1991; Farley & Gonzalez, 1996; Ferris, Louie, & Farley, 1998; He, Kram, & McMahon, 1991; McMahon, Valiant, & Fredrick, 1987; McMahon & Cheng, 1990). The model consists of a point mass supported by a single massless linear “leg spring.”

The main mechanical parameter studied when using the spring-mass model is the stiffness of the leg spring, defined as the ratio of the maximal force in the spring to the maximum leg compression at the middle of the stance phase (Farley & Gonzalez, 1996). Moreover, although it does not correspond to any physical spring, the term vertical stiffness is used to describe the vertical motion of the center of mass (CM) during contact (Farley & Gonzalez, 1996; McMahon & Cheng, 1990), and is defined as the ratio of the maximal force to the vertical displacement of the CM as it reaches its lowest point, i.e., the middle of the stance phase. Maximal ground reaction force and CM displacement measurements during running are needed to measure these parameters requiring dynamometers such as overground or treadmill-mounted force platforms, or even video motion-analysis (e.g., Arampatzis, Brüggemann, & Metzler, 1999). Such equipment is costly and not practical for field measurements.

Considering the descriptive and predictive power of the spring-mass model in running mechanics and the serious technical issues mentioned, the aim of this study was to propose and validate a simple calculation method for assessing leg and vertical stiffness during running. The computations inherent in the proposed method are based on a few simple mechanical parameters: contact and flight times, forward running speed, leg length, and body mass. We validated the method by comparing leg and vertical stiffness values measured with a force platform to those obtained using the proposed method during treadmill and overground running.

Methods

For practical reasons our study was divided into two different protocols, one aiming to validate the presented method during treadmill running, the other during overground running. However, in both cases the force platform and sine-wave method stiffness values were each calculated in the same way. All participants gave their informed consent to take part in those protocols.

Treadmill running: Eight young men (age 24 ± 2 yrs, height 1.78 ± 0.07 m, body mass 76.0 ± 7.0 kg; mean \pm SD) volunteered to participate in this study. They were physical education students and experienced in treadmill running. After a 5-min warm-up at $3.33 \text{ m}\cdot\text{s}^{-1}$, they performed 30-s running bouts at 3.33, 3.89, 4.44, 5, 5.56, 6.11, and $6.67 \text{ m}\cdot\text{s}^{-1}$ at their preferred step frequency (separated by 2 min of rest) on a treadmill dynamometer (HEF Techmachine, Andrézieux-Bouthéon, France). Using the same calibration procedure as Belli, Bui, Berger, Geysant, and Lacour (2001), we determined the treadmill's static nonlinearity to be less than 0.5% and 1%, respectively, in the vertical and horizontal directions. Natural vibration frequency (treadmill hit with a hammer) were 147 Hz in the vertical direction and 135 Hz in the anterior-posterior and mediolateral directions. Vertical ground reaction forces, belt velocity, and flight and contact times were measured at a sampling frequency of 500 Hz. All values were averaged for 10 consecutive steps for each velocity.

Overground running: 10 young men (age 23 ± 3 yrs, height 1.80 ± 0.05 m, body mass 66.4 ± 5.3 kg) who were elite middle-distance runners volunteered to run over a 10-m force platform (Kistler, Winterthur, Switzerland; natural frequency higher than 150 Hz, horizontal to vertical cross-talk lower than 2%) at 4, 5, 6, and 7 $\text{m}\cdot\text{s}^{-1}$ and at their maximal running velocity. Velocity was measured by two pairs of photocells placed at each end of the force platform. Vertical ground reaction forces and flight and contact times were measured for one step at each velocity at a sampling rate of 1.8 kHz.

Reference method. Vertical stiffness: The vertical stiffness k_{vert} (in $\text{kN}\cdot\text{m}^{-1}$) was calculated as:

$$k_{\text{vert}} = F_{\text{max}} \cdot \Delta y_c^{-1} \tag{1}$$

with F_{max} the maximal ground reaction force during contact (in kN) and Δy_c the vertical displacement of the CM when it reaches its lowest point (in m), determined by double integration of the vertical acceleration over time, as proposed by Cavagna (1975). Figure 1 shows a typical example of force and vertical displacement of the CM evolutions during contact.

Leg stiffness: The stiffness of the leg spring k_{leg} (in $\text{kN}\cdot\text{m}^{-1}$) was calculated as follows:

$$k_{\text{leg}} = F_{\text{max}} \cdot \Delta L^{-1} \tag{2}$$

with ΔL the peak displacement of the leg spring (in m) calculated from the initial leg length L (great trochanter to ground distance in a standing position), running velocity v (in $\text{m}\cdot\text{s}^{-1}$), and the contact time t_c (in s), as per Farley and Gonzales (1996; Eq. 2 and 3):

$$\Delta L = L - \sqrt{L^2 - \left(\frac{vt_c}{2}\right)^2} + \Delta y_c \tag{3}$$

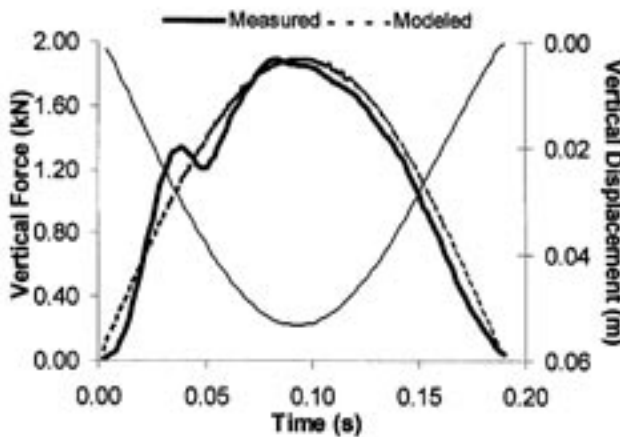


Figure 1 — Typical measured (thick line) and modeled (dotted line) vertical force over time during contact on treadmill running, and corresponding downward vertical CM displacement (thin line) (velocity of $5 \text{ m}\cdot\text{s}^{-1}$, body mass of 68 kg).

Sine-wave method. Model proposed: This method is based on a model used by Dalleau et al. (2004) for vertical jumps, which considers the force as a function of time during contact to be a simple sine function (Alexander, 1989; Kram & Dawson, 1998):

$$F(t) = F_{\max} \sin\left(\frac{\pi}{t_c} t\right) \quad (4)$$

The validity of this postulate was checked by comparing areas under the measured and modeled $F(t)$ curves for all steps analyzed (Figure 1). The mean absolute bias was calculated as $|(Modeled - Force\ platform)/Force\ platform| \times 100$.

Calculations: Complete calculations and assumptions are shown in the Appendix section of this paper.

The modeled vertical stiffness \hat{k}_{vert} was calculated as the ratio of the modeled maximal force \hat{F}_{\max} over the modeled vertical CM displacement $\Delta\hat{y}_c$:

$$\hat{k}_{\text{vert}} = \hat{F}_{\max} \cdot \Delta\hat{y}_c^{-1} \quad (5)$$

$$\text{with } \hat{F}_{\max} = mg \frac{\pi}{2} \left(\frac{t_f}{t_c} + 1\right) \quad (6)$$

m being the participant's body mass (in kg), and t_f and t_c , respectively, being the flight and contact times (in s)

$$\text{and } \Delta\hat{y}_c = \frac{\hat{F}_{\max} t_c^2}{m \pi^2} + g \frac{t_c^2}{8} \quad (7)$$

The modeled leg stiffness \hat{k}_{leg} was calculated as follows:

$$\hat{k}_{\text{leg}} = \hat{F}_{\max} \cdot \Delta\hat{L}^{-1} \quad (8)$$

$$\text{with } \Delta\hat{L} = L - \sqrt{L^2 - \left(\frac{vt_c}{2}\right)^2} + \Delta\hat{y}_c \quad (9)$$

the modeled leg length variation (in m).

The participant's leg length was also modeled as: $L_{\text{mod}} = 0.53h$ where h is the participant's height (in m), according to the anthropometric equations of Winter (1979), in order to check the validity of the presented stiffness calculation method using leg length values obtained from participant's height data.

In order to quantify the force platform-model values difference, we calculated an absolute mean error bias as follows: $\text{Bias} = |(Modeled - Force\ platform)/Force\ platform| \times 100$. The force platform-model relationships were further described by means of linear regressions and calculation of the determination coefficient R^2 .

We then performed an ANOVA with the Scheffé post hoc test in order to check the eventual effect of velocity on the above-mentioned force platform-model bias. The statistically significant level was set at 0.05.

Results

The bias between force platform and modeled $F(t)$ areas was 5.33% (ranging from 11.7% at 3.33 m·s⁻¹ to 1.7% at 6.67 m·s⁻¹) for treadmill running, 542 steps analyzed, and 2.93% (ranging from 3.17% at 4 m·s⁻¹ to 2.33% at 7 m·s⁻¹) for overground running, 50 steps analyzed.

Table 1 Main Mechanical Parameters Measured With Reference Method During Treadmill Running ($M \pm SD$)

	Reference	Modeled	Bias (%)
Δy_c (m)	0.05 ± 0.01	0.05 ± 0.02	3.28 ± 1.10
ΔL (m)	0.20 ± 0.03	0.20 ± 0.03	0.93 ± 0.43
F_{\max} (kN)	2.05 ± 0.34	1.91 ± 0.32	6.93 ± 2.52
k_{vert} ($\text{kN}\cdot\text{m}^{-1}$)	37.70 ± 8.84	37.74 ± 8.87	0.12 ± 0.53
k_{leg} ($\text{kN}\cdot\text{m}^{-1}$)	10.37 ± 2.34	9.75 ± 2.19	6.05 ± 3.02

Note: Measured with reference method, corresponding values calculated with proposed method and error bias values in between. Values presented are averaged for all participants and all velocities.

Table 2 Main Mechanical Parameters Measured With Reference Method During Overground Running ($M \pm SD$)

	Reference	Modeled	Bias (%)
Δy_c (m)	0.05 ± 0.01	0.05 ± 0.01	2.34 ± 2.42
ΔL (m)	0.16 ± 0.02	0.16 ± 0.02	0.67 ± 1.09
F_{\max} (kN)	2.13 ± 0.21	2.06 ± 0.24	3.24 ± 2.08
k_{vert} ($\text{kN}\cdot\text{m}^{-1}$)	51.39 ± 21.46	50.21 ± 20.40	2.30 ± 1.63
k_{leg} ($\text{kN}\cdot\text{m}^{-1}$)	13.28 ± 1.85	12.95 ± 2.13	2.54 ± 1.16

Note: Measured with reference method, corresponding values calculated with proposed method and error bias values in between. Values presented are averaged for all participants and all velocities.

The values of the different mechanical parameters measured and modeled and the reference-model bias are shown in Tables 1 and 2 for treadmill and overground running, respectively. For the vertical and leg stiffness, we obtained reference-model error biases of 0.12% (ranging from 1.53% at $6.67 \text{ m}\cdot\text{s}^{-1}$ to 0.07% at $6.11 \text{ m}\cdot\text{s}^{-1}$) and of 6.05% (from 9.82% at $3.33 \text{ m}\cdot\text{s}^{-1}$ to 3.88% at $6.67 \text{ m}\cdot\text{s}^{-1}$), respectively, during treadmill running. During overground running the bias was 2.30% (from 3.64% at $5 \text{ m}\cdot\text{s}^{-1}$ to 0.25% at $6 \text{ m}\cdot\text{s}^{-1}$) for the vertical stiffness, and 2.54% (from 3.71% at $5 \text{ m}\cdot\text{s}^{-1}$ to 1.11% at maximal velocity) for leg stiffness.

Further, the reference-model linear regressions were significant ($p < 0.01$; $R^2 = 0.89$ to 0.98) for both vertical and leg stiffness either on the treadmill or during overground running (Figures 2 and 3, respectively).

The changes in reference and modeled values of stiffness with running velocities are reported in Figure 4 for treadmill and overground running. The ANOVA and the Scheffé post hoc test demonstrated for treadmill running a significant dif-

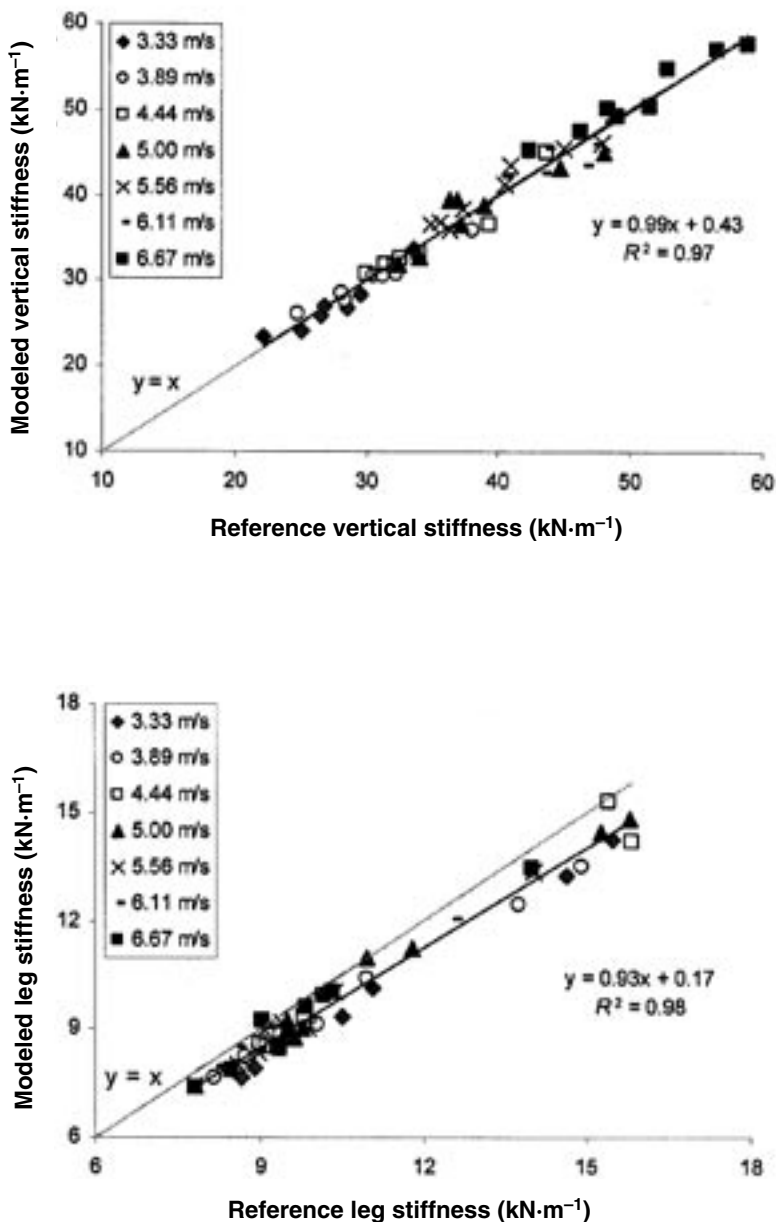


Figure 2 — Significant reference-model linear regressions ($p < 0.01$) obtained during treadmill running for vertical stiffness (upper panel) and leg stiffness (lower panel) compared to the identity line. Each dot represents a mean value of stiffness for a participant at the corresponding velocity.

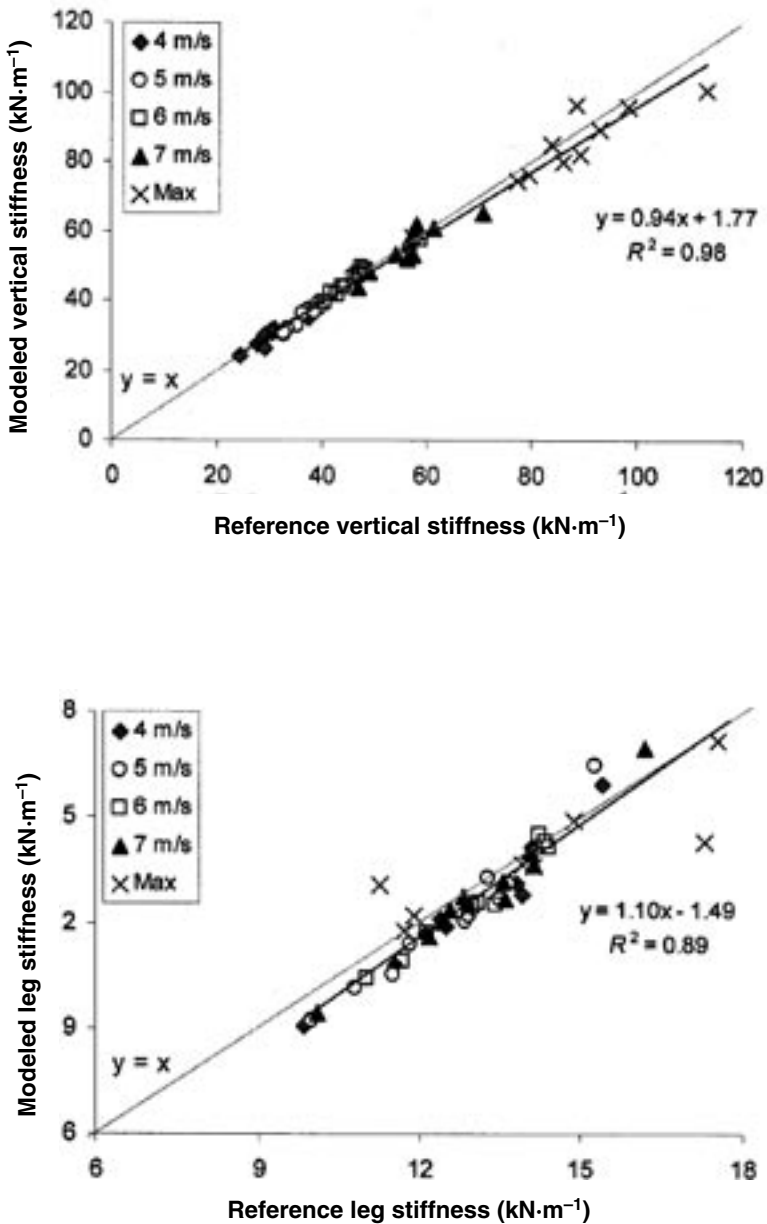


Figure 3 — Significant reference-model linear regressions ($p < 0.01$) obtained during overground running for vertical stiffness (upper panel) and leg stiffness (lower panel) compared to the identity line. Each dot represents the value of stiffness of the step analyzed for each participant at the corresponding velocity.

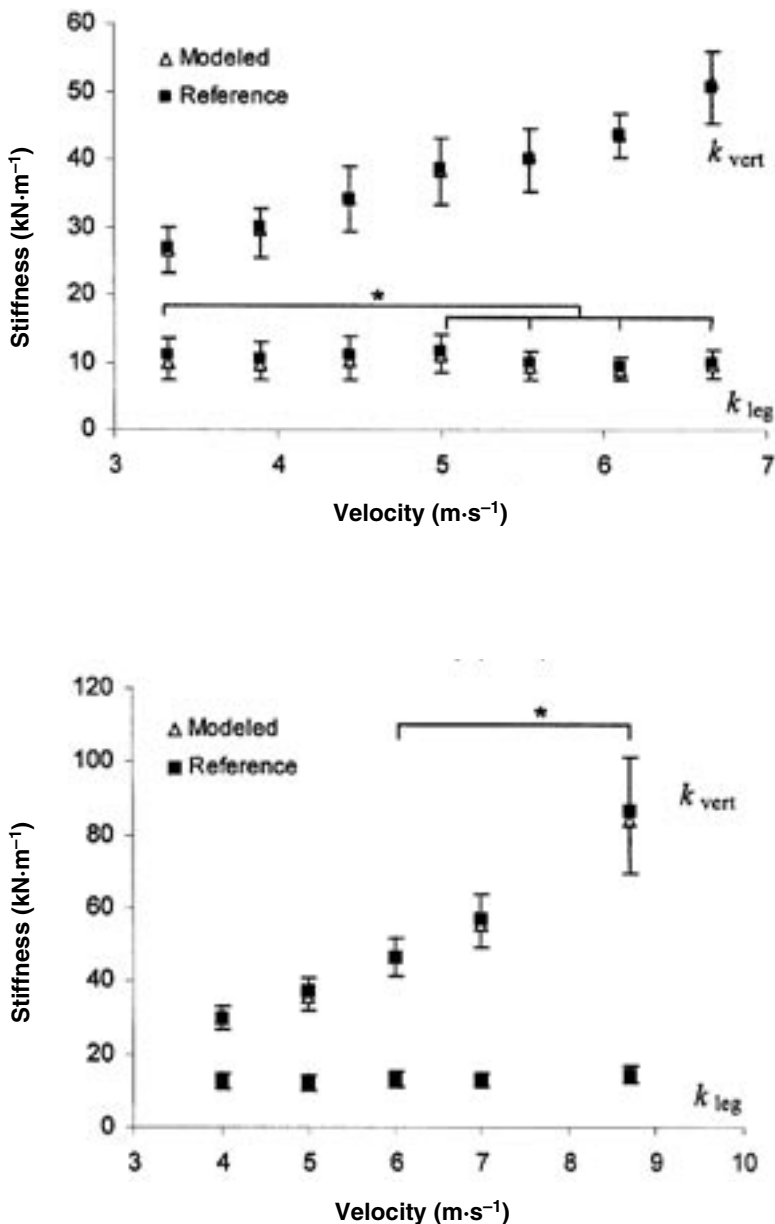


Figure 4 — Changes in the modeled and reference vertical and leg stiffness with running velocity during treadmill running (upper panel) and overground running (lower panel). *Significant ($p < 0.05$) difference in reference-model bias obtained by ANOVA and Scheffé post hoc test.

ference ($p < 0.05$) between the bias in leg stiffness assessment at $3.33 \text{ m}\cdot\text{s}^{-1}$ and that obtained at the four highest velocities. For overground running, a significant difference was observed on the vertical stiffness bias ($p < 0.05$) obtained at $6 \text{ m}\cdot\text{s}^{-1}$ and at the maximal velocity (Figure 4).

The leg length calculation using Winter's equation led to a mean error bias of $1.94 \pm 1.51\%$, and the linear regression between the measured leg length and that obtained using Winter's model was significant ($R^2 = 0.89$; $p < 0.01$).

Discussion

The values of the different mechanical parameters obtained in this study—vertical ground reaction forces, displacements, and stiffness—are in line with those of studies about the spring-mass model in running (Arampatzis et al., 1999; Blickhan, 1989; Farley & Gonzales, 1996; Ferris et al., 1998; He et al., 1991; McMahon et al., 1987; McMahon & Cheng, 1990). It should be noted that we obtained slight differences in displacements and vertical and leg stiffness between our two experimental protocols (treadmill and overground running), due to the different running abilities of the two populations in these two parts of the study.

Indeed, highly trained middle-distance runners who performed the force platform protocol showed lower CM displacement and leg length change and higher vertical and leg stiffness than the nonspecialists who volunteered to perform the treadmill protocol (e.g., leg stiffness: $10.37 \pm 2.34 \text{ kN}\cdot\text{m}^{-1}$ for the treadmill protocol group vs. $13.28 \pm 1.85 \text{ kN}\cdot\text{m}^{-1}$ for overground runners). This is in line with the study of Mero and Komi (1986), who observed higher “apparent spring constants” of the support leg and lower CM displacement during running in highly trained sprinters compared to less-skilled ones (Luhtanen & Komi, 1980) or marathon runners (Ito, Komi, Sjödin, Bosco, & Karlsson, 1983).

The stiffness vs. velocity patterns (Figure 4), i.e., the increase in vertical stiffness and the constancy of leg stiffness with increasing velocity either on the treadmill or in overground conditions, has been previously noted in the literature (He et al., 1991; McMahon et al., 1987).

Validity of the proposed sine-wave method: The aim of this study was to provide a calculation method based on a few simple parameters allowing vertical and leg stiffness to be assessed during treadmill and overground running without a force platform. The low bias obtained between force platform and model values (from 0.12% to 6%), and the high determination coefficients (from 0.89 to 0.98, $p < 0.01$), demonstrate the validity of this calculation method during both treadmill and overground running. Furthermore, the validity of this method was tested for a wide range of velocities (i.e., from 3.33 to $6.67 \text{ m}\cdot\text{s}^{-1}$ on the treadmill and $4 \text{ m}\cdot\text{s}^{-1}$ to maximal velocity on the force platform) with runners of different ability levels, from nonspecialists to highly trained middle-distance runners, all giving acceptable results (Figures 2 and 3). This may allow us to use the proposed method during submaximal to maximal velocity running, with either nonspecialists or elite athletes, in field conditions. It should be also noted that the mechanical parameter input to the model (maximal force, CM displacement, leg length change) also showed acceptable reference-model bias (0.67 to 6.93%, Tables 1 and 2).

Basis postulate and assumptions: The basis postulate of this study was that the $F(t)$ curve can be fitted by means of a simple sine function. The validity of this sine modeling, recently used in a study aimed at validating a stiffness measuring

method during hopping (Dalleau, Belli, Viale, Lacour, & Bourdin, 2004), was checked by comparing areas under $F(t)$ curves for all steps analyzed on the treadmill and on the force plate, and we obtained low error bias values (5.33% and 2.93%, respectively). On the treadmill the accuracy of this sine modeling was improved at faster velocities, the bias ranging from 11.7% at $3.33 \text{ m}\cdot\text{s}^{-1}$ to only 1.7% at $6.67 \text{ m}\cdot\text{s}^{-1}$. This was probably due to the alteration of the passive impact peak in the vertical ground reaction force at low velocity and to an $F(t)$ curve more closely approximating a sine curve at faster velocities. This could explain the significant effect of velocity on reference-model bias in leg stiffness observed between that obtained at $3.33 \text{ m}\cdot\text{s}^{-1}$ and that at the four highest velocities (Figure 4, upper panel), the accuracy of the basic sine-wave postulate increasing at faster velocities on the treadmill. To the contrary, this velocity effect on the bias was not observed during overground running.

The bias in $F(t)$ curve fitting by the sine function was rather constant, and the previously mentioned velocity effect on reference-model bias was only significant ($p = 0.043$) in vertical stiffness between that obtained at $6 \text{ m}\cdot\text{s}^{-1}$ and that at the maximal velocity (Figure 4, lower panel). This smaller effect of velocity may be explained by the fact that $F(t)$ curves of elite level “forefoot striking” runners showed smaller or no passive impact peaks, whatever the velocity, with shorter contact times and $F(t)$ plot shapes closer to a sine-wave (Nilsson & Thorstensson, 1989).

Sensitivity analysis: In order to further determine the influence of the different mechanical parameters constituting the presented model on the vertical and leg stiffness calculated, we performed a sensitivity analysis (Figure 5). It was then possible to observe the relative influence of each mechanical parameter on the calculated stiffness values. It should be noted that **the most sensitive parameter, for both vertical and leg stiffness estimates, is the contact time.** Its variation influences the stiffness in a proportion of about 1 to 2, i.e., for instance a 10% reduction in contact time leads, according to this model, to a 20% increase in vertical stiffness or even 25% in leg stiffness. All the other parameters have a 1 : 1 weight or even less, especially anthropometrical parameters of body mass and leg length.

Therefore, although the legs are not stiff either at landing or takeoff, resulting in a slight overestimation of leg length (Arampatzis et al., 1999), the influence of such a phenomenon on the accuracy of the stiffness calculations was not important. In addition, the results of the present study showed that the leg length value used in this sine-wave model can be obtained using anthropometric equations, on the basis of the individual’s height, without significant changes in the stiffness values obtained. Furthermore, a recent study did not find any significant variation in the stiffness values obtained with the proposed method, using an estimated value of leg length from the participant’s height according to Winter’s model (Winter, 1979), showing that this parameter is not crucial for improving the accuracy of the sine-wave model (Jeannin, 2003).

This model also assumes a constant point-of-force application on the ground during the entire contact phase. However, the location of the point-of-force application was shown to move forward by about 0.16 m (Lee & Farley, 1998), and this constitutes another limitation of the simplest spring-mass model.

Finally, it is worth noting that the limits of the proposed sine-wave method are also those inherent in the use of any theoretical spring-mass model, i.e., **the human lower limb is not a true linear spring in a physical sense.** This point has been widely discussed elsewhere (Blickhan, 1989; Farley & Gonzales, 1996; Ferris, Louie, &

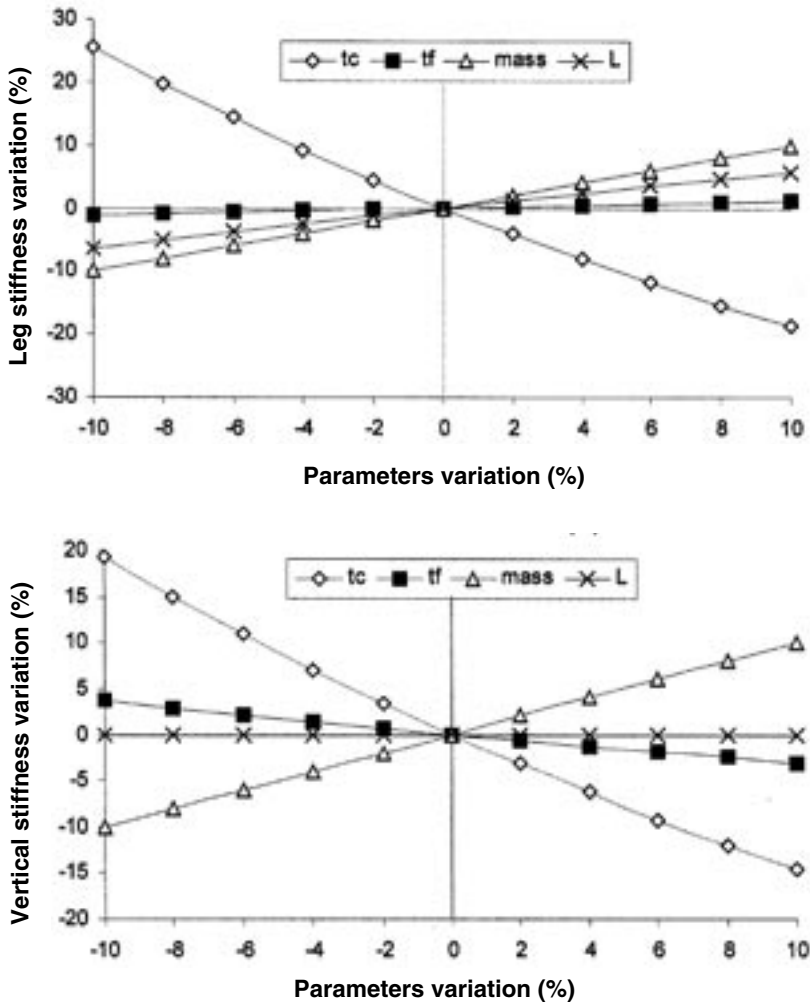


Figure 5 — Sensitivity analysis: Variations of the different mechanical parameters constituting the equation’s model plotted against the corresponding leg stiffness (upper panel) and vertical stiffness (lower panel) variations.

Farley, 1998; He et al., 1991; McMahon & Cheng, 1990). Even if one prefers the term “quasi-stiffness” (Latash & Zatsiorsky, 1993), the simplicity and predictive power of the spring-mass model in studies aimed at understanding and analyzing human running performance outweigh the limitations.

The equations based on a force-time curve sine modeling allow these calculations from simple mechanical parameters of flight and contact times, leg length, body mass, and running velocity. The reference-model biases on calculated stiffness were acceptable for a wide range of velocities, different levels of runners, and

either on a treadmill dynamometer or overground. To conclude, this new method may allow researchers to measure and to understand the influence of leg stiffness on performance in field running conditions and also to improve track & field coaching and training.

References

- Alexander, R.M. (1989). On the synchronization of breathing with running in wallabies (*Macropus spp.*) and horses (*Equus caballus*). *Journal of Zoology (London)*, **218**, 69-85.
- Alexander, R.M. (1992). A model of bipedal locomotion on compliant legs. *Philosophical Transactions of the Royal Society of London*, **B338**, 189-198.
- Arampatzis, A., Brüggemann, G.-P., & Metzler, V. (1999). The effect of speed on leg stiffness and joint kinetics in human running. *Journal of Biomechanics*, **32**, 1349-1353.
- Belli, A., Bui P., Berger, A., Geysant, A., & Lacour, J.-R. (2001). A treadmill ergometer for three-dimensional ground reaction forces measurement during walking. *Journal of Biomechanics*, **34**, 105-112.
- Blickhan, R. (1989). The spring-mass model for running and hopping. *Journal of Biomechanics*, **22**, 1217-1227.
- Cavagna, G.A. (1975). Force platforms as ergometers. *Journal of Applied Physiology*, **39**, 174-179.
- Cavagna, G.A., Heglund, N.C., & Taylor, C.R. (1977). Mechanical work in terrestrial locomotion: Two basic mechanisms for minimizing energy expenditure. *American Journal of Physiology*, **233**, R243-R261.
- Cavagna, G.A., Saibene, F.P., & Margaria, R. (1964). Mechanical work in running. *Journal of Applied Physiology*, **19**, 249-256.
- Dalleau, G., Belli, A., Bourdin, M., & Lacour, J.-R. (1998). The spring-mass model and the energy-cost of treadmill running. *European Journal of Applied Physiology*, **77**, 257-263.
- Dalleau, G., Belli, A., Viale, F., Lacour, J.-R., & Bourdin, M. (2004). A simple method for field measurement of leg stiffness in hopping. *International Journal of Sports Medicine*, **25**, 170-176.
- Farley, C.T., Blickhan, R., Saito, J., & Taylor, C.R. (1991). Hopping frequency in humans: A test of how springs set stride frequency in bouncing gaits. *Journal of Applied Physiology*, **71**, 2127-2132.
- Farley, C.T., & Gonzalez, O. (1996). Leg stiffness and stride frequency in human running. *Journal of Biomechanics*, **29**, 181-186.
- Ferris, D., Louie, M., & Farley, C.T. (1998). Running in the real world: Adjusting leg stiffness for different surfaces. *Proceedings of the Royal Society of London*, **265**, 898-994.
- He, J., Kram, R., & McMahon, T. (1991). Mechanics of running under simulated low gravity. *Journal of Applied Physiology*, **71**, 863-870.
- Ito, A., Komi, P.V., Sjödin, B., Bosco, C., & Karlsson, J. (1983). Mechanical efficiency of positive work in running at different speeds. *Medicine and Science in Sports and Exercise*, **15**, 299-308.
- Jeannin, T. (2003). *Mesure des paramètres mécaniques de la course sur tapis roulant et sur le terrain* [Measurement of mechanical parameters during treadmill and overground running]. Saint-Etienne, France: University of Saint-Etienne.
- Kram, R., & Dawson, T.J. (1998). Energetics and biomechanics of locomotion by red kangaroos (*Macropus rufus*). *Comparative Biochemistry and Physiology*, **120**, 41-49.
- Latash, M.L., & Zatsiorsky, V.M. (1993). Joint stiffness: Myth or reality? *Human Movement Studies*, **12**, 653-692.

- Lee, C., & Farley, C.T. (1998). Determinants of the center of mass trajectory in human walking and running. *Journal of Experimental Biology*, **201**, 2935-2944.
- Luhtanen, P., & Komi, P.V. (1980). Force-, power-, and elasticity-velocity relationships in walking, running and jumping. *European Journal of Applied Physiology*, **44**, 279-289.
- McMahon, T.A., & Cheng, G.C. (1990). The mechanics of running: How does stiffness couple with speed? *Journal of Biomechanics*, **23**, 65-78.
- McMahon, T.A., Valiant, G., & Frederick, E.C. (1987). Groucho running. *Journal of Applied Physiology*, **62**, 2326-2337.
- Mero, A., & Komi, P.V. (1986). Force-, EMG-, and elasticity-velocity relationships at sub-maximal, maximal and supramaximal running speeds in sprinters. *European Journal of Applied Physiology*, **55**, 553-561.
- Nilsson, J., & Thorstensson, A. (1989). Ground reaction forces at different speeds of human walking and running. *Acta Physiologica Scandinavica*, **136**, 217-227.
- Winter, D.A. (1979). *Biomechanics of human movement*. New York: Wiley & Sons.

APPENDIX

Modeled vertical stiffness computations

The modeled vertical stiffness \hat{k}_{vert} was calculated as:

$$\hat{k}_{\text{vert}} = \hat{F}_{\text{max}} \cdot \Delta \hat{y}_c^{-1} \quad (5)$$

with \hat{F}_{max} the modeled maximal force and $\Delta \hat{y}_c$ the modeled vertical peak displacement of the center of mass (CM) during contact.

\hat{F}_{max} computations

The pattern of vertical ground reaction force over time was modeled using the following equation:

$$F(t) = F_{\text{max}} \cdot \sin\left(\frac{\pi}{t_c} \cdot t\right) \quad (4)$$

with F_{max} the peak force value and t_c the contact time.

From this equation, the momentum change during contact is:

$$\int_0^{t_c} [F(t) - mg] \cdot dt = m\Delta u = mgt_f \quad (\text{A10})$$

with m the participant's body mass, u the vertical velocity, g the gravity acceleration, and t_f the mean flight time (mean of flight times before and after contact). Substituting Eq. A4 in Eq. A10 gives:

$$\int_0^{t_c} \left[F_{\text{max}} \cdot \sin\left(\frac{\pi}{t_c} \cdot t\right) - mg \right] \cdot dt = m\Delta u = mgt_f \quad (\text{A11})$$

$$\left[-F_{\text{max}} \frac{t_c}{\pi} \cos\left(\frac{\pi}{t_c} \cdot t\right) \right]_0^{t_c} - mgt_c = mgt_f \quad (\text{A12})$$

$$2 F_{\text{max}} \frac{t_c}{\pi} mg (t_f + t_c) \quad (\text{A13})$$

The modeled peak force during contact is then obtained as:

$$\hat{F}_{\text{max}} = mg \frac{\pi}{2} \left(\frac{t_f}{t_c} + 1 \right) \quad (6)$$

$\Delta \hat{y}_c$ computations

Based on Eq. A4 and according to the fundamental law of dynamics, vertical velocity is obtained by integrating the vertical acceleration of the CM during contact:

$$u(t) = \int_{t_0}^{t_c} \left[\frac{F(t)}{m} - g \right] \cdot dt + u(t_0) \quad (\text{A14})$$

$u(t_0)$ being the downward vertical velocity at the beginning of contact.

$$u(t) = \int_{t_0}^{t_c} \left[\frac{\hat{F}_{\max}}{m} \sin \left(\frac{\pi}{t_c} \cdot t \right) - g \right] \cdot dt + u(t_0) \quad (\text{A15})$$

$$u(t) = \left[-\frac{\hat{F}_{\max}}{m} \frac{t_c}{\pi} \cos \left(\frac{\pi}{t_c} \cdot t \right) \right]_{t_0}^{t_c} - gt + u(t_0) \quad (\text{A16})$$

Knowing that the vertical velocity is nil at the time of half-contact:

$$u\left(\frac{t_c}{2}\right) = \frac{\hat{F}_{\max}}{m} \frac{t_c}{\pi} - g \frac{t_c}{2} + u(t_0) = 0 \quad (\text{A17})$$

$$\frac{\hat{F}_{\max}}{m} \frac{t_c}{\pi} + u(t_0) = g \frac{t_c}{2} \quad (\text{A18})$$

The final expression of vertical velocity during contact being:

$$u(t) = -\frac{\hat{F}_{\max}}{m} \frac{t_c}{\pi} \cos \left(\frac{\pi}{t_c} \cdot t \right) - gt + g \frac{t_c}{2} \quad (\text{A19})$$

Integrating this vertical velocity over time, vertical displacement can be obtained:

$$y(t) = \int_{t_0}^{t_c} u(t) \cdot dt + y(t_0) \quad (\text{A20})$$

with $y(t_0)$ the vertical position of the CM at the beginning of contact.

Assuming $y(t_0) = 0$ and substituting Eq. A19 in Eq. A20:

$$y(t) = \int_{t_0}^{t_c} \left[-\frac{\hat{F}_{\max}}{m} \frac{t_c}{\pi} \cos \left(\frac{\pi}{t_c} \cdot t \right) - gt + g \frac{t_c}{2} \right] \cdot dt \quad (\text{A21})$$

$$y(t) = \left[-\frac{\hat{F}_{\max}}{m} \frac{t_c^2}{\pi^2} \sin \left(\frac{\pi}{t_c} \cdot t \right) - \frac{1}{2} gt^2 \right]_{t_0}^{t_c} + g \frac{t_c}{2} t \quad (\text{A22})$$

$$y(t) = -\frac{\hat{F}_{\max}}{m} \frac{t_c^2}{\pi^2} \sin \left(\frac{\pi}{t_c} \cdot t \right) - \frac{1}{2} gt^2 + g \frac{t_c}{2} t \quad (\text{A23})$$

The total CM displacement at the time of half contact, i.e., for $t = t_c / 2$ is then:

$$\Delta \hat{y}_c = -\frac{\hat{F}_{\max}}{m} \frac{t_c^2}{\pi^2} + g \frac{t_c^2}{8} \quad (7)$$

Modeled leg stiffness computations

The modeled leg stiffness \hat{k}_{leg} was calculated as:

$$\hat{k}_{\text{leg}} = \hat{F}_{\max} \cdot \Delta \hat{L}^{-1} \quad (8)$$

with $\Delta \hat{L}$ the modeled leg peak displacement during contact.

$\Delta \hat{L}$ computations

$\Delta \hat{L}$ was obtained on the basis of the spring-mass model's typical equations and assumptions (Farley & Gonzales, 1996; McMahon & Cheng, 1990):

$$\Delta \hat{L} = L - \sqrt{L^2 - \left(\frac{v t_c}{2} \right)^2} + \Delta \hat{y}_c \quad (9)$$

L being the participant's leg length and v the constant horizontal velocity.

Copyright of Journal of Applied Biomechanics is the property of Human Kinetics Publishers, Inc. and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.